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This publication is designed to educate high school students about the problems and challenges of transportation. The parent project was conceived as a mathematics curriculum project, but developed into an interdisciplinary project to ensure that students would become well informed regarding the transportation decisions they make. In the activities in this booklet, students learn how to select a route through a network of streets. The central problem concerns looking for perfect newspaper delivery routes, i.e., routes that go down each street just once without any backtracking. The puzzles require students to experiment with several routes in order to find a pattern. By carefully analyzing their experimentations to determine why a newspaper carrier would get stuck consistently at certain intersections, students gain an understanding of why the pattern exists. Students then design their own perfect routes based on their findings. (PVD)

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ED 425 062

TRANSPORTATION DEMAND MANAGEMENT

Choosing a Delivery Route

STUDENT'S EDITION

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Preface

The Transportation Demand Management documents began to take shape in 1993, when the California Department of Transportation (Caltrans) and the California Department of Education undertook a joint project to educate the students of California about the problems and challenges of transportation. The project was conceived as a mathematics curriculum project and then developed into an interdisciplinary attempt to ensure that students would become well informed regarding the transportation decisions they make, now and in the future.

The Department of Education recruited a team of university and high school mathematicians, Caltrans personnel, and professionals from the business community to develop the four mathematics units for high school students: *Transportation Choices, Data Analysis, Downtown Distances,* and *Choosing a Delivery Route.*

Working from the mathematics units, educators in history–social science, English–language arts, environmental education, and industrial technology took on the task of making meaningful connections between their disciplines and the transportation issues discussed in the mathematics documents. The two documents they produced are *Problems, Issues, and Public Policy Choices: A History–Social Science Resource Unit* and *Investigations and Problem Analysis: A Science Resource Unit, Grades Eight Through Twelve.*

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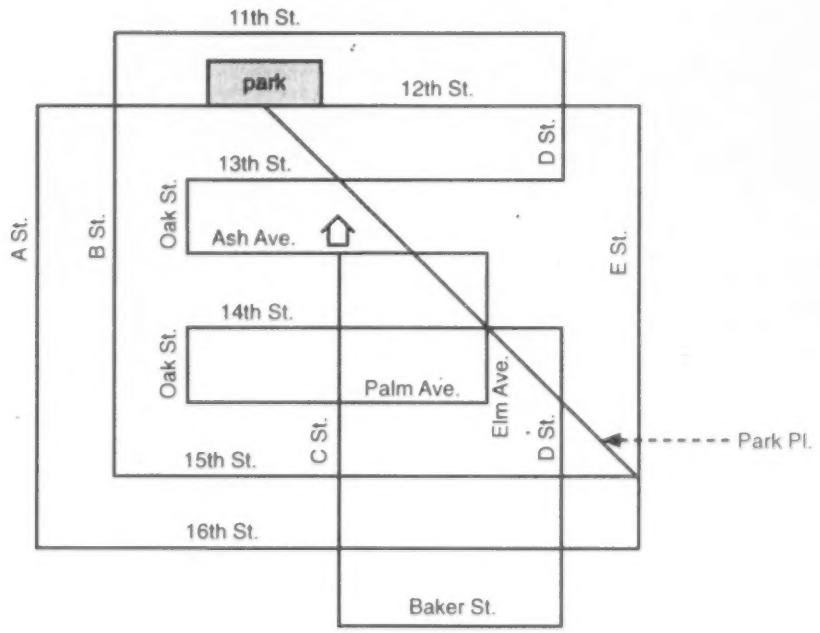
Choosing a Delivery Route

In this section, you will learn how to select a route through a network of streets. The central problem that is posed concerns a newspaper delivery route; but you will see that the method you develop is useful in situations other than just delivery routes. Even more important than the method you develop for solving the problems is the thought process you go through to solve them: You are given a puzzle and, first, you experiment with it. With enough experimentation, a pattern appears to you that seems to solve the puzzle. By looking carefully at your experimentation, and possibly asking more questions, you understand why the pattern is there and you know that you have solved the puzzle.

You will find frequent questions and exercises highlighted throughout the text. It is important to stop reading and respond to each question before you continue reading. Your teacher will tell you what form those answers should take—written or oral.

Looking for a Perfect Route

Pat has a job delivering papers in the neighborhood. Almost every house on every street subscribes to the newspaper. As Pat rides his bike down the streets, he delivers papers to houses on both sides of the street. It occurred to Pat that he would save time in delivering papers if he could find a way to deliver the papers and go down each street just once—without any backtracking—and wind up back at his house. He calls this a **perfect route**. Pat has not been able to find a perfect route, and he wondered whether it is because there is no such route or because he just hasn't been able to find it yet. A map of Pat's neighborhood is shown below.



Pat's Neighborhood

Pat lives at the corner of Ash Ave. and C St.

Pat wonders if there is a route that starts and ends at his house and doesn't go along any street twice but does go down every street.

1. Pat's Route

Try to find such a route. Use a marking pen on one of the copies of the map that you have been given to keep track of which streets have already been covered in your route. Compare what you found with what other members of your group found. If someone in your group finds a route for Pat, make sure that everyone in the group agrees that it is a perfect route.

Pat was unable to find a perfect route; so he concluded that there must not be a perfect route.

2. Pat's Conclusion

In your group, discuss whether Pat's conclusion is justified.

Here's another approach to take in trying to find a perfect route. Start at Pat's house and try to find a perfect route by seeing how far he could go without any backtracking. Use a marking pen on one of the copies of the map that you have been given to keep track of which streets have already been covered in your route. Don't stop until Pat has nowhere to go without covering a street more than once; but be sure to stop the route when Pat can go no farther.

3. Pat's Stopping Place

Make a note of where Pat was forced to stop on this route. Each of the members of your group should try this approach and compare the routes that you found. Were any of you able to find a perfect route—one that covered each street once and only once and that allowed Pat to end at his home? Were there any similarities in the routes that you found? At which different places on the map did the routes stop? Discuss with your group what you found. Try to find an explanation for what you found.

Pat actually tried this method of finding a perfect route and got stuck at Park Place and 12th St. A friend of his, Marina, who was sitting in her front yard at the corner of Park Place and 14th St., watched him ride up and away, then again watched him ride up and away, then again ride up and away.

4. Marina's Observations

Why wasn't Pat forced to stop at Marina's house?

Another friend of his, Marcus, who was at the park at the corner of Park Place and 12th St., also watched him ride up and away and then ride up again and stop.

5. Marcus's Observations

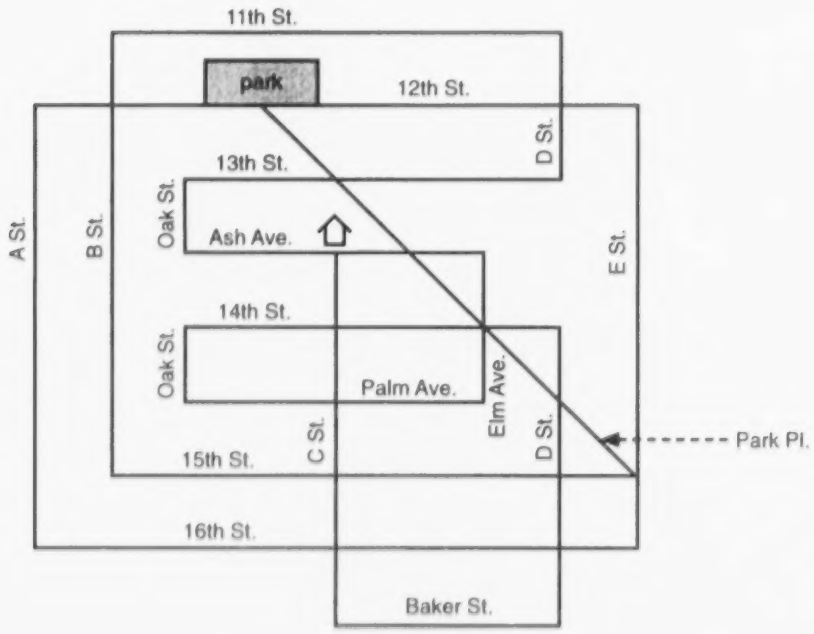
If Marcus were at the park and watching Pat take your route, what would he see? Why was Pat forced to stop at the park?

Imagine friends of Pat's at each intersection watching him come and go on this route.

6. Where Pat Doesn't Stop

What would they see? Why wasn't Pat forced to stop at any of these other intersections? Work with your group to try to find a way to explain what you have found.

Exercises



Pat's Neighborhood

1. Write an explanation to someone who isn't in this class, describing why all the attempts to find a perfect route ended up at the park.
2. Suppose that Pat started his route at the park. Where would he end up when he was done with his route?
3. Suppose that Pat doesn't have to deliver newspapers to any houses on Park Place and doesn't want to ride his bike on Park Place at all. If he starts out from his house and tries to cover as many streets as he can without backtracking or riding on Park Place, where will he get stuck? How could you tell this without even trying to find a route?

Exploring Transit Numbers

In another neighborhood, Anna is looking for a perfect route for her newspaper deliveries. A map of the streets to which she delivers papers is shown below.

Anna's Neighborhood

Anna lives at the corner of Broadway and Peach. She starts out on her route from her house and tries to see how far she can get without backtracking.

Try the same thing that Anna did and see where you finally get stuck. Use a marking pen on one of the copies of the map that you have been given to keep track of which streets have already been covered in your route. Start at Anna's house and don't stop until you have nowhere to go without covering a street more than once. But be sure to stop the route when you can go no farther.

1. Anna's Stopping Place

Make a list of the intersections where members of your group get stuck.

2. Transit Numbers

At each intersection on the map, note the number of ways you could enter or leave that intersection. For example, at the intersection of Peach and Bay, you could go north on Peach, south on Peach, west on Bay, or east on Bay. There are four ways out of the intersection, so you should write a number 4 by that intersection.

We'll call the number of ways in or out of an intersection the **transit number** for the intersection.

3. Transit Numbers for Stopping Places

Look at the intersections where members of your group got stuck when they tried the previous exercise. Make a list of the transit numbers of those intersections. What do you notice about those transit numbers?

Think about an intersection that has transit number 4, such as Euclid and Plum. If Anna's friend were standing at that corner, she might never see Anna if Anna never got to that intersection. Or she might see Anna enter the intersection, then leave, and not see her again if Anna got stuck elsewhere after that. Or she might see Anna enter the intersection, then leave, then enter again, then leave again.

4. Why Anna Gets Stuck

Why couldn't Anna possibly be stuck (that is, have nowhere to go without backtracking) at Euclid and Plum?

Think of any intersection that has a transit number that is even (such as Grape and Pascal, which has transit number 2; or Broadway and Plum, which has transit number 4). Could Anna get stuck at that intersection? Illustrate with a diagram.

Think of any intersection that has a transit number that is odd, other than Broadway and Peach (such as Apple and Market, which has transit number 3; or Pascal and Pine, which has transit number 5). Could Anna get stuck at that intersection? Illustrate with a diagram.

Notice that Anna's house is at Broadway and Peach, which has an odd transit number. Could Anna possibly get stuck at Broadway and Peach? Illustrate with a diagram.

Exercises

4. Write a paragraph to explain to someone who isn't in this class what you learned today about finding routes that don't require backtracking. Be sure to explain the connection between the transit number and whether or not Anna might get stuck at that intersection.
5. Draw a map of your immediate neighborhood, including at least eight streets. Don't make the map cover too large an area. It is best to start by deciding upon boundary streets for your neighborhood, then filling in the streets inside those boundaries with no streets extending beyond the boundaries. Write the transit numbers for each intersection. Think of a dead end on a street as an intersection with transit number 1. Use these transit numbers to answer this question: Could you start at your house on a bike, ride down every street on your map exactly once, and never leave the streets on your map?
6. Design and draw a map that doesn't have a perfect route.
7. Redesign the street layout in Anna's neighborhood (by adding streets) so that she can find a perfect route.

Learning About Getting Stuck

In each of the two maps that we considered, the person who delivered the paper lived at an intersection that had an odd transit number. Now consider a person who lives at an intersection with an even transit number. Using the map of Anna's neighborhood, suppose that she lives at the corner of Euclid and Apple.

Anna's Neighborhood

1. Starting at Euclid and Apple

Where could Anna get stuck on her route if she keeps going until she can't go anywhere without riding on a street twice? Before you begin marking a map to see where she might get stuck, try using the same kind of analysis that you used before to explain where she gets stuck.

Now each member of your group should use a marking pen on one of the copies of the map that you have been given to keep track of which streets have already been covered in your route. Start at Anna's house and don't stop until you have nowhere to go without covering a street more than once; but be sure to stop the route when you can go no farther.

2. Stopping Places

Where did the others in your group get stuck? Is that what you anticipated?

3. Start and Stop at Euclid and Apple

How could Anna be stuck at Euclid and Apple even though it has an even transit number? Try to explain the problem in terms of Anna's entering and leaving this intersection.

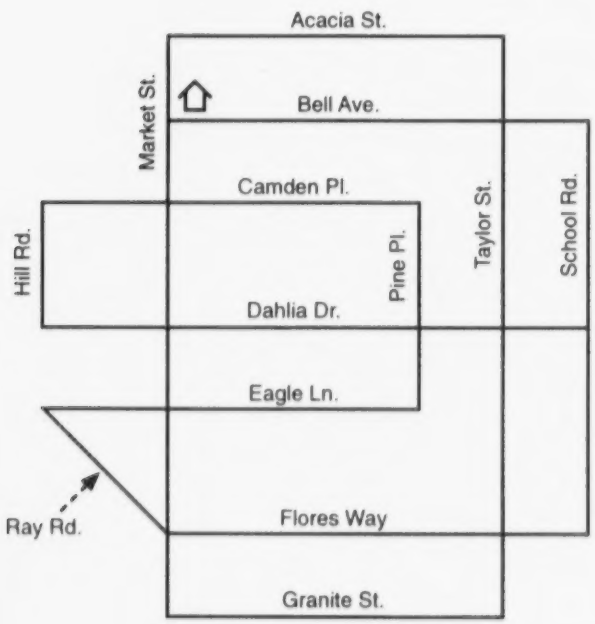
In some ways, it seems as though you may be close to finding a perfect route for Anna because the route begins and finishes at her house without ever going down any street twice. The reason this isn't a perfect route is that, as you can see, some streets in her neighborhood didn't get newspapers. Having made some progress, we want to see if there is anything we can do to find a perfect route. Suppose that some people think that they have found a perfect route for Anna.

4. Euclid and Cherry

If you were sitting on a porch at the corner of Euclid and Cherry, what would you see if Anna followed this perfect route?

Exercises

8. Is there anywhere that Anna could live in her neighborhood that would allow her to find a perfect route for delivering papers? Explain.



Anthony's Neighborhood

9. Refer to the map shown above. If Anthony lives at the corner of Market and Bell, where will he get stuck delivering papers (with no place to go other than to go down a street for a second time) if he starts at his house? Why is that the only place where he would get stuck?

10. If Carla lives at the corner of Eagle and Pine in Anthony's neighborhood, where might she get stuck delivering papers (with no place to go other than to go down a

street for a second time) if she starts at her house? Why are these the only three places where she could get stuck?

11. According to the map of Anthony's neighborhood, is there anywhere that someone with a paper route could live that would allow that person to find a perfect route for delivering to the neighborhood? If so, where would the person live, and what would the perfect route be? If not, why not?

Finding a Pattern in Transit Numbers

We've seen that if there are any intersections that have an odd transit number, then no perfect route can be found. If there are two intersections with odd transit numbers, then it appears that we can find a route that starts at one of those two intersections, covers every street exactly once, and ends up at the other intersection that has an odd transit number. That might be useful if the delivery person lives at one of those intersections and plans to deliver papers on the way to school, which is at the other intersection that has an odd transit number. But what would happen if there were just one intersection with an odd transit number? It seems that if you started a paper route at this intersection, there would be nowhere to get stuck—not even at the intersection where you started. This example seems to mean that you would go on delivering papers forever. Of course, that doesn't make any sense. So let's try to make an example of a map that has just one intersection with an odd transit number.

1. One Odd Transit Number

Try to make an example of a map that has just one intersection with an odd transit number. Construct a small map—one with no more than 8 or 9 intersections. Next to each intersection, write its transit number. Were you successful in constructing a map with just one intersection with an odd transit number?

2. Counting Even and Odd

Make a map with no more than ten intersections and count the number of intersections that have an even transit number and those that have an odd transit number. What pattern do you see in what you and your classmates found?

3. Add the Transit Numbers

Now count the sum of all the transit numbers of all the intersections on your map. What pattern do you see in what you and your classmates found?

Now count the total number of street sections in your map. A street section means a section of street between two adjacent intersections.

4. Count the Street Sections

What pattern do you see in what you and your classmates found? Write a description of the pattern you found and an explanation of why that pattern occurs.

5. Even Sum of Transit Numbers

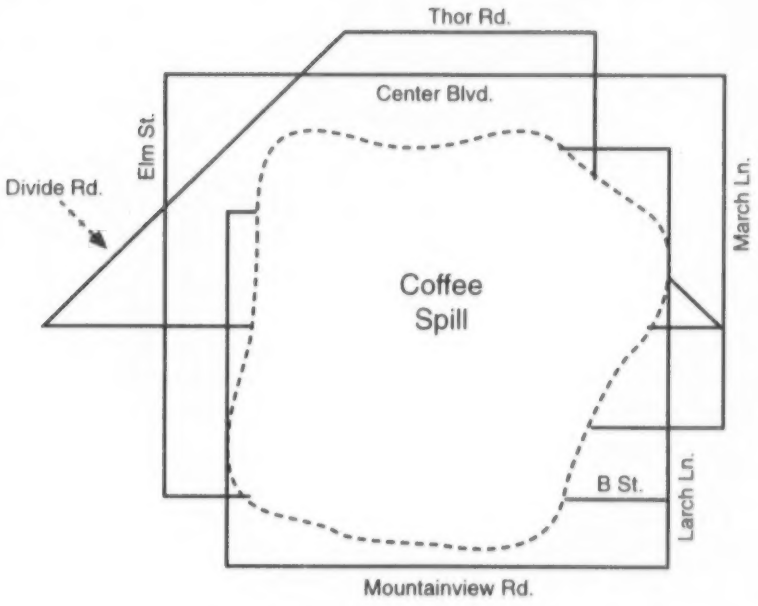
Now that you have seen this pattern, write an explanation of why the sum of the transit numbers is always even.

We started this section by trying to find a map that has just one intersection with an odd transit number.

6. How Many Odd Transit Numbers?

If you were able to construct such a map, then what could you say about the sum of the transit numbers on that map? In general, if you add up a series of even numbers and an odd number of odd numbers, would the result be even or odd? What implication does this finding have for maps and transit numbers?

Exercises



Mai's Neighborhood

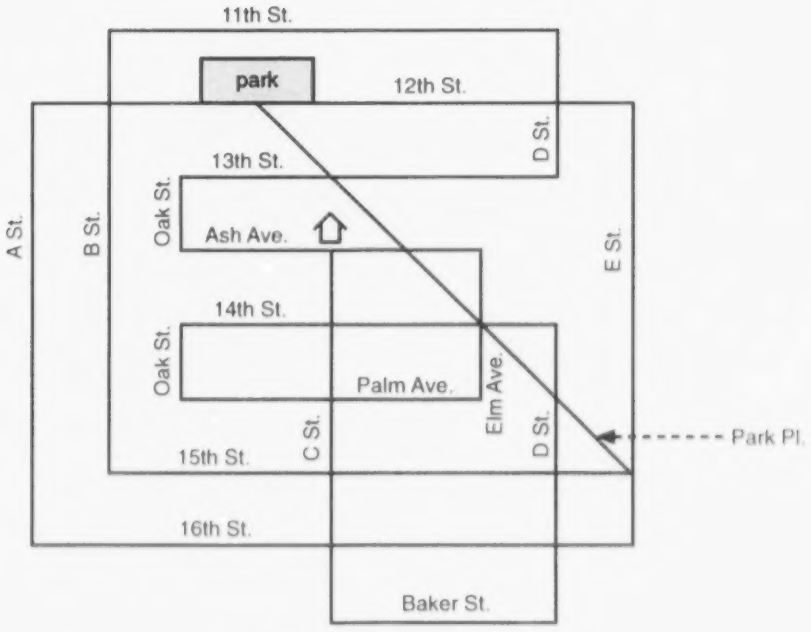
12. Shown above is a map of Mai's neighborhood, to which she delivers papers. Mai has been unable to find a perfect route for delivering papers. In fact, when she starts at the intersection in front of her house, no matter what route she takes, she always gets stuck at the same intersection and has to backtrack to finish her route and get back home. You will notice that a portion of the map is missing—coffee was spilled on it. What information can you provide about where Mai lives, and where she gets stuck on her route, by looking at the map?

Developing the Best Possible Route

In practice, almost any neighborhood has lots of intersections with an odd transit number. For example, any T intersection has transit number 3. And for our purposes we must think of a dead end on a street as an intersection with transit number 1. Thus it is quite unlikely that you would find a paper route for which there is a perfect route.

On the other hand, many streets on which papers are delivered are so busy or so wide that a paper carrier would want to ride down the street twice—once on each side of the street. Therefore, those busy streets should be thought of as being two streets because the paper carrier wants to go down those streets exactly twice. You can get a more realistic idea of whether a neighborhood has a perfect route by making a map of the neighborhood with double lines for any street that is so wide or busy that you need to make two trips to cover both sides of the street.

But the realistic question is not really whether a perfect route exists, because it is unlikely that a perfect route exists. In realistic situations, the question is apt to be what route can the paper carrier take that involves as little backtracking as possible. We'll call this a **best possible route**—a route that starts and finishes at the same point, covers every street at least once, but is as short as possible. Consider the following map of Pat's neighborhood:



Pat's Neighborhood

Recall that Pat lives at the corner of Ash and C. You probably found a route for Pat that covers every street but leaves him at Park and 12th instead of at his house. So he must backtrack to get home, which he would do by the shortest route. If you counted how many times Pat entered or left any intersection on the map, the number would always be even. When he was stuck at Park and 12th, he had entered or left that intersection three times; but when he leaves again for home (now backtracking), it becomes four times, an even number.

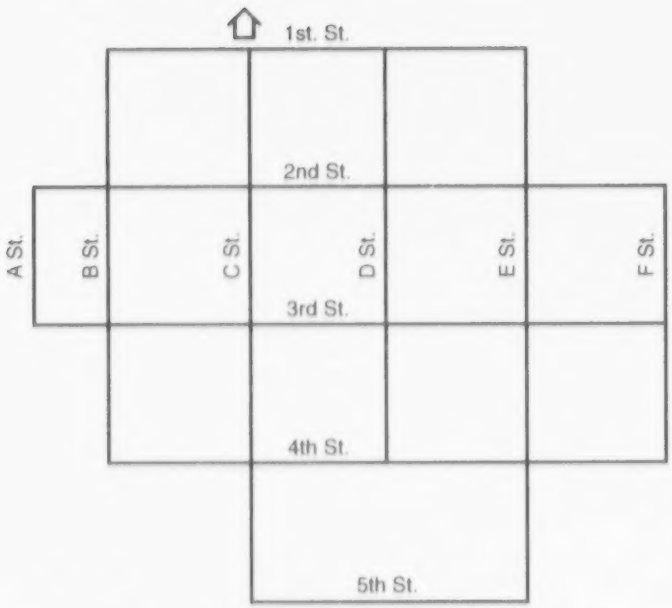
To find a best possible route when there are intersections with an odd transit number, first identify the intersections with an odd transit number. There is always an even number of them. Now look for ways to pair these intersections so that the routes connecting each pair of intersections with odd transit numbers are as short as possible. For example, refer to the following map of Anna's neighborhood:

Anna's Neighborhood

We identify four intersections with odd transit numbers: Broadway and Peach (where Anna lives), Market and Apple, Euclid and Cherry, and Pascal and Apple. We will use the block as our unit of measurement. For example, the distance from Euclid and Apple to Euclid and Peach is 1 block, and the distance from Market and Apple to Market and Plum is 2 blocks. We will pair up the four intersections with odd transit numbers as follows: Market and Apple with Pascal and Apple, and Broadway and Peach with Euclid and Cherry. The distance from Market and Apple to Pascal and Apple is 4 blocks, and the distance from Broadway and Peach to Euclid and Cherry is 3 blocks, making a total of 7 blocks. If we pair up Broadway and Peach with Pascal and Apple (for a distance of 4 blocks) and Euclid and Cherry with Market and Apple (for a distance of 5 blocks), we get a total of 9 blocks. Since the first pairing is shorter, the first way of pairing is better than the second.

To use this method to find the best possible route, decide first how you want the intersections paired up; then connect those intersections by doubling the streets between the paired intersections. This doubling is really just a note to yourself that these streets will be covered twice somewhere along the route. With these streets doubled, you have a map in which each intersection effectively has an even transit number. So you can now look for your route.

Exercises



Steve's Neighborhood

13. On the map shown above, find a best possible route for Steve, who lives at the corner of 1st and C.

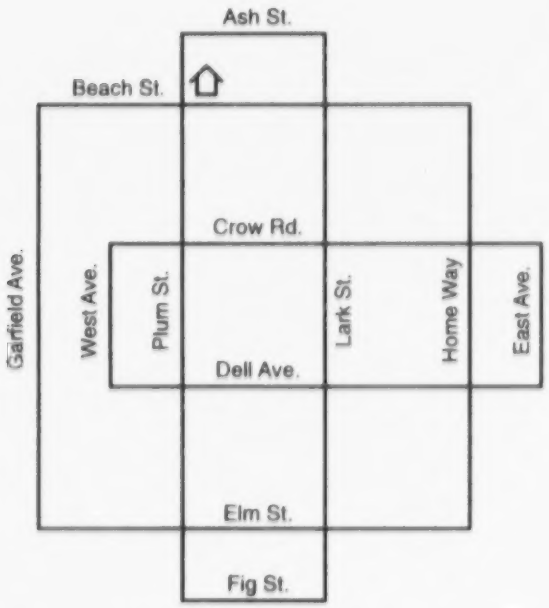
14. On the map shown above, find a best possible route for Jennifer, who lives at the corner of 2nd and F.

Understanding the Conditions for a Perfect Route

Suppose that you have a map with no intersections with odd transit numbers. We have not yet seen why there should be a perfect route.

1. Myrna's Route

On the following map, find a perfect route for Myrna, who lives at Plum and Beach.



Myrna's Neighborhood

2. Finding Good Routes

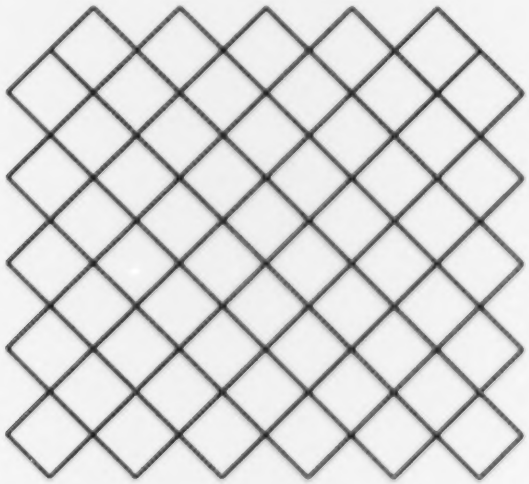
Explain how you can be sure that whenever there are no intersections with odd transit numbers, there will always be a perfect route.

Cultural Connections

The Bushoong people of Zaire have the role of decorators among the Kuba chiefdom. When a Bushoong chief is enthroned, he proclaims a particular design as his official sign. The Kuba children play games drawing these figures in the sand. As they draw them in the sand, they do so without lifting their finger from the sand and without retracing any segment of the drawing.

Bushoong Design

Find a way to draw the following Bushoong figure in the sand without lifting your finger or retracing a segment of the figure. Also explain how this problem is essentially the same as the newspaper delivery problem that we have been considering.

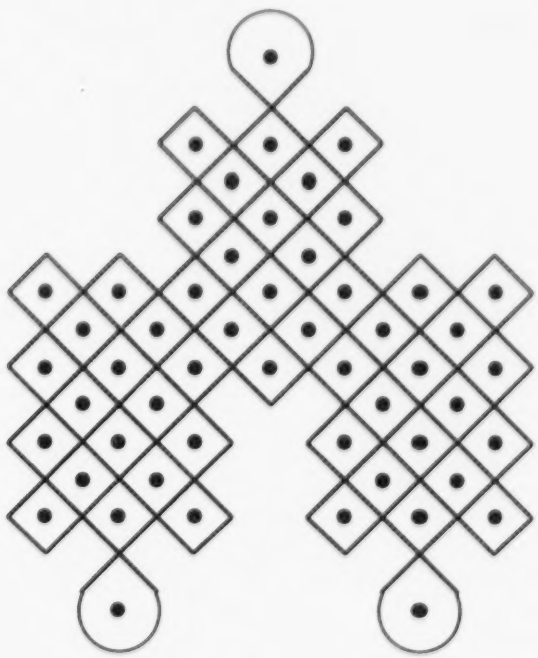


Cultural Connections

The Tshokwe people of Angola use designs that are traced in the sand as a part of a storytelling tradition. The figures they trace are called sona, which are traced by the storytellers as the story is told. The dots in the figures represent people or trees or other significant aspects of the story being told. The figure that is traced around these dots is done without lifting the finger and without retracing a segment of the figure.

Tshokwe Design

Find a way to draw the following Tshokwe figure in the sand without lifting your finger or retracing a segment of the figure. Also explain how this problem is essentially the same as the newspaper delivery problem that we have been considering.

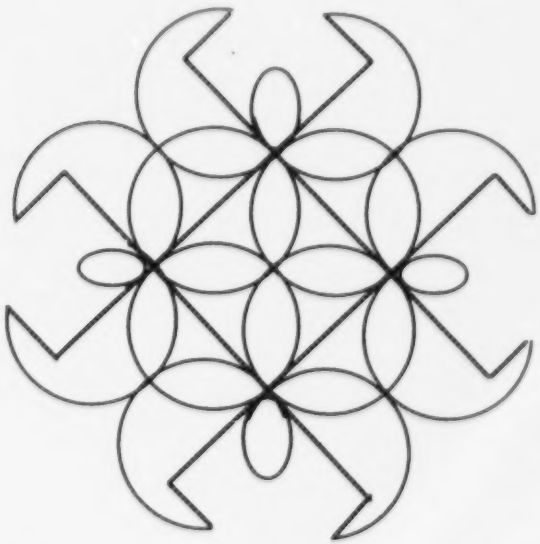


Cultural Connections

In the Republic of Vanuatu in the South Pacific, the Malekula also trace figures in the sand for several cultural reasons. For example, the Malekula believe that passage to the Land of the Dead requires the ability to trace certain figures in the sand.

Malekula Design

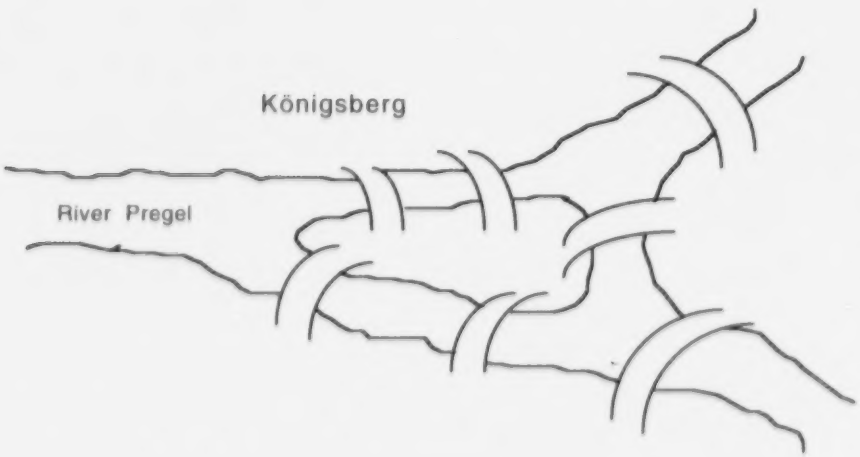
Find a way to draw the following Malekula figure in the sand without lifting your finger or retracing a segment of the figure.



Cultural Connections

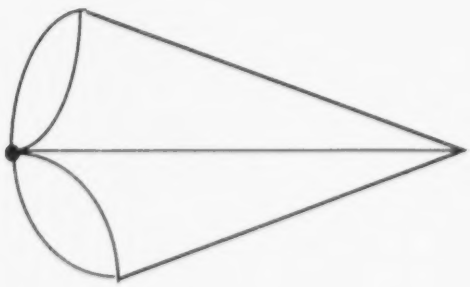
One of the most interesting aspects of mathematics is the way in which it bridges cultures. In this example, mathematics does not seem to be a part of other cultures so much for its utility, which is usually the case, as it is for other reasons.

It appears to have occurred in one European culture as a game. In the town of Königsberg in the 1700s, there was a question of whether it was possible to find a way to take a walk that crossed each of the seven bridges in the town exactly once. Königsberg lies on the river Pregel, and within the town limits there is an island. A crude map of the town is shown below:



No one had been able to find a route to walk through the town crossing every bridge exactly once. But, of course, that is not enough to allow us to conclude that there is no such route. The mathematician Leonhard Euler (pronounced *oiler*) recognized the mathematical nature of the problem. With the correct observation, you can see that the so-called Königsberg bridge problem is the same as the newspaper delivery problem. The bridges become like the streets in the newspaper delivery problem, and the land masses become like

the street intersections. Then the map of Königsberg becomes like the street map shown below:



1. **Königsberg Maps**

   Label the seven bridges in the map of Königsberg and label the corresponding seven streets in the street map. Label the four land masses in the map and the corresponding four intersections in the street map.

2. **Königsberg Problem**

   Using the street map, explain why it is not possible to find a route through Königsberg that goes over each bridge exactly once.

Euler was also able to solve this problem as you have by making a map. Today the study of perfect routes on maps is known as Eulerian circuits in honor of Leonhard Euler. This area of mathematics has applications to such problems as deciding routes for snow plows in clearing streets; for letter carriers; and for inspection of networks, such as pipelines.

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